30 points | Quiz 5 Math 112 | PLEDGED Key

You may use the reference sheet for this class but you may not consult another person. You pledge that this quiz represents your work, completed in one sitting. Use your paper to respond the questions below.

1. Evaluate. Label each step carefully. (6 points each, label each step carefully)

a) $\int_0^{\infty} \frac{e^{3x}}{e^{6x}+4}dx$ $\quad \frac{\pi}{12} - \frac{1}{6}\text{Arctan}\left(\frac{1}{2}\right)$

b) $\int_{2/\pi}^{\infty} \frac{\sin(x^{-1})}{x^2}dx$ $\quad 1$

c) $\int_{-\infty}^{0} e^x \cos(2x)dx$ $\quad \frac{1}{5}$

2. Sketch the graph and find the area inside $r = 5\cos(5\theta)$. Clearly label all important aspects. (12 points)

$\frac{25\pi}{4}$

1. a) $\displaystyle\int_0^\infty \frac{e^{3x}}{e^{6x}+4}\,dx \Rightarrow \lim_{k\to\infty}\int_0^k \frac{e^{3x}}{e^{6x}+4}\,dx$

Aside:

$\displaystyle\int \frac{e^{3x}}{e^{6x}+4}\,dx$ $\quad u = e^{3x}$ $\quad du = 3e^{3x}\,dx$ $\quad \frac{1}{3}\,du = e^{3x}\,dx$ $\quad \displaystyle\frac{1}{3}\int \frac{du}{u^2+4}$

$\frac{1}{3}\cdot\frac{1}{2}\text{Arctan}\,\frac{u}{2} + C$

$\frac{1}{6}\text{Arctan}\,\frac{e^{3x}}{2} + C$

$\displaystyle\lim_{k\to\infty}\left[\frac{1}{6}\text{Arctan}\,\frac{e^{3x}}{2}\right|_0^k = \lim_{k\to\infty}\frac{1}{6}\,\text{Arctan}\left(\frac{e^{3k}}{2}\right) - \frac{1}{6}\text{Arctan}\,\frac{e^0}{2} =$

$\frac{1}{6}\cdot\frac{\pi}{2}$

$\frac{1}{6}\cdot\frac{\pi}{2} - \frac{1}{6}\text{Arctan}\left(\frac{1}{2}\right)$

b) $\displaystyle\int_{2/\pi}^\infty \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx \rightarrow \lim_{k\to\infty}\int_{2/\pi}^k \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx$

Aside: $\displaystyle\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx$ $\quad u = x^{-1}$

$du = -x^{-2} \rightarrow -du = \frac{1}{x^2}\,dx$

$-\displaystyle\int \sin u\,du = \cos u + C$

$\cos\left(\frac{1}{x}\right) + C$

$\displaystyle\lim_{k\to\infty}\left(\cos\left(\frac{1}{x}\right)\right|_{2/\pi}^k = \lim_{k\to\infty}\cos\left(\frac{1}{k}\right) - \cos\left(\frac{\pi}{2}\right)$

$\cos(0) = 1$

$1 - 0 = \boxed{1}$

c) $\int_{-\infty}^{0} e^x \cos(2x)\,dx \rightarrow \lim_{k\to-\infty} \int_{k}^{0} e^x \cos(2x)\,dx$

Aside: $\int e^x \cos(2x)\,dx$ $\quad u = e^x \quad dv = \cos(2x)\,dx$
$\quad du = e^x\,dx \quad v = \frac{\sin(2x)}{2}$

$$\int e^x \cos(2x)\,dx = \frac{e^x \sin(2x)}{2} - \frac{1}{2}\int e^x \sin(2x)\,dx$$

$u = e^x \quad dv = \sin(2x)\,dx$
$du = e^x\,dx \quad v = -\frac{\cos(2x)}{2}$

$$= \frac{e^x \sin(2x)}{2} - \frac{1}{2}\left(-\frac{e^x\cos(2x)}{2}\right) - \frac{1}{2}\left(+\int \frac{e^x\cos(2x)}{2}\,dx\right)$$

$$\frac{5}{4}\int e^x \cos(2x)\,dx = \frac{e^x \sin(2x)}{2} + \frac{e^x\cos(2x)}{4} - \cancel{\frac{1}{4}\int e^x\cos(2x)\,dx}$$

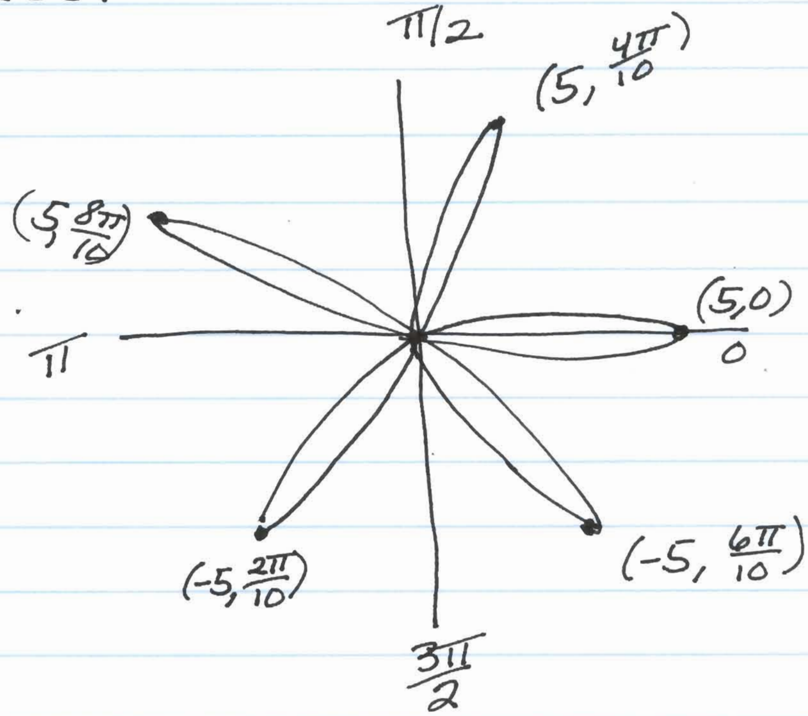
$$\int e^x \cos(2x)\,dx = \frac{2}{5}e^x \sin(2x) + \frac{1}{5}e^x\cos(2x) + C$$

$$\lim_{k\to-\infty}\left(\frac{2}{5}e^x \sin(2x) + \frac{1}{5}e^x\cos(2x)\right)\Bigg|_k^0 = \frac{2}{5}e^0\cancelto{0}{\sin(0)} + \frac{e^0}{5}\cos(0)$$

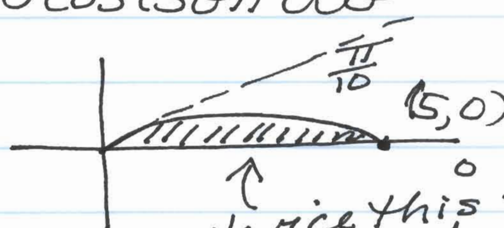
$$- \lim_{k\to-\infty} \cancelto{0}{\frac{e^k}{5}}\left(2\sin(2k) + \cos(2k)\right) = \frac{1}{5} - 0 = \boxed{\frac{1}{5}}$$

2. $r = 5\cos(5\theta)$

r	θ
5	0
0	$\frac{\pi}{10}$
-5	$\frac{2\pi}{10}$
0	$\frac{3\pi}{10}$
5	$\frac{4\pi}{10}$
0	$\frac{5\pi}{10}$
-5	$\frac{6\pi}{10}$
0	$\frac{7\pi}{10}$
5	$\frac{8\pi}{10}$
0	$\frac{9\pi}{10}$
-5	π



ONE LEAF } $A = 2 \cdot \frac{1}{2} \int_0^{\pi/10} (5\cos(5\theta))^2 \, d\theta$



twice this for one leaf

Rose:

$$5 \cdot 25 \int_0^{\pi/10} \cos^2(5\theta)\, d\theta$$

5 leaves

$$\frac{125}{2} \int_0^{\pi/10} [1 + \cos(10\theta)]\, d\theta$$

$$\frac{125}{2}\left[\theta + \frac{1}{10}\sin(10\theta)\right]_0^{\pi/10} =$$

$$\frac{125}{2} \cdot \frac{\pi}{10} + \cancel{\frac{125}{2} \cdot \frac{1}{10} \sin\pi}^{\,0} - \left(\cancel{\frac{125}{5} \cdot 0}^{\,0} + \cancel{\frac{125}{2} \cdot \frac{1}{10} \sin(0)}^{\,0}\right)$$

$$\frac{125\pi}{20} = \frac{25\pi}{4}$$