30 points | Math 111 | PLEDGED Key

Quiz 3 | Print name

You may not use any materials on this quiz. You may not consult another person or even discuss this quiz until it is turned in for grade. You must work through these problems in <u>one sitting</u>. Your signature on the pledged line above certifies that you have complied with the rules for taking this quiz.

Date/time began______________________ date/time ended______________________

1. Find the derivatives of the following. Simplify appropriately: (4 points each, 16 points)

a. $f(x) = \tan^2\sqrt{x} \quad -\sec^2\sqrt{x}$

$\tan^2\sqrt{x} + 1 = \sec^2\sqrt{x}$

$\tan^2\sqrt{x} - \sec^2\sqrt{x} = -1$

So $\boxed{f'(x) = 0}$

OR

$f'(x) = \cancel{2}\tan\sqrt{x}(\sec^2\sqrt{x})(\frac{1}{\cancel{2}}x^{-\frac{1}{2}})$

$- \cancel{2}\sec\sqrt{x}(\sec\sqrt{x}\tan\sqrt{x})(\frac{1}{\cancel{2}}x^{-\frac{1}{2}})$

$$f'(x) = \frac{\tan\sqrt{x}\sec^2\sqrt{x}}{\sqrt{x}} - \frac{\sec^2\sqrt{x}\tan\sqrt{x}}{\sqrt{x}} = 0$$

b. $g(x) = (\ln\sin^2 x)^3$

$$g'(x) = 3(\ln\sin^2 x)^2\left(\frac{1}{\sin^{\cancel{2}} x}\cdot 2\cancel{\sin x}\cdot\cos x\right)$$

$$\boxed{g'(x) = 6(\ln\sin^2 x)^2\cot x}$$

c. $h(x) = \ln\left(\frac{x^2e^{x^2}}{(x^2-2)^2}\right) = \ln x^2 + \ln e^{x^2} - \ln(x^2-2)^2$

$h(x) = 2\ln x + x^2 - 2\ln(x^2-2)$

$$h'(x) = \frac{2}{x} + 2x - \frac{2}{x^2-2}(2x)$$

$$\boxed{h'(x) = \frac{2}{x} + 2x - \frac{4x}{x^2-2}}$$

1. continued . . .

d. $y = \cos(\tan^3(4x))$

$$y' = -\sin(\tan^3(4x))\left[3\tan^2(4x)\right](\sec^2(4x))(4)$$

$$\boxed{y' = -12\sin(\tan^3(4x))\tan^2(4x)\sec^2(4x)}$$

2. Will there be horizontal tangent lines for the following functions? If so, give the ordered pairs; if not, explain clearly why not: (4 points each, 8 points)

$y' = 0$ where ?

a. $y = \dfrac{x^2 - 3x}{x - 1}$

$$y' = \frac{(x-1)(2x-3) - (x^2-3x)(1)}{(x-1)^2} = \frac{2x^2 - 5x + 3 - x^2 + 3x}{(x-1)^2}$$

$$y' = \frac{x^2 - 2x + 3}{(x-1)^2}$$

Solve: $x^2 - 2x + 3 = 0$

$$x = \frac{2 \pm \sqrt{4 - 4(3)}}{2} = \frac{2 \pm \sqrt{-8}}{2}$$

No real number solutions

So no points where the tangent line is horizontal

b. $y = (x-4)^{2/3}(x-1)^{1/3}$

$$y' = (x-4)^{2/3} \cdot \frac{1}{3}(x-1)^{-2/3} + (x-1)^{1/3} \cdot \frac{2}{3}(x-4)^{-\frac{1}{3}}$$

$$y' = \frac{(x-4)^{2/3}}{3(x-1)^{2/3}} + \frac{2(x-1)^{1/3}}{3(x-4)^{1/3}} = \frac{x - 4 + 2x - 2}{3(x-1)^{2/3}(x-4)^{1/3}}$$

$$y' = \frac{3x - 6}{3(x-1)^{2/3}(x-4)^{1/3}} = \frac{\cancel{3}(x-2)}{\cancel{3}(x-1)^{2/3}(x-4)^{1/3}}$$

$$y' = \frac{x-2}{(x-1)^{2/3}(x-4)^{1/3}}$$

$y' = 0$ at $x = 2$

$$\boxed{(2, \sqrt[3]{4})}$$

Note:

At $x = 2$: $y = (2-4)^{2/3}(2-1)^{1/3}$

$= (-2)^{2/3} \cdot 1 = \sqrt[3]{4}$

3. Given: $f(x) = \dfrac{1}{\sqrt{25-x}}$

a) Use the definition of derivative to find the derivative. Show all steps clearly. (4 points)

$$\lim_{\Delta x \to 0} \frac{\dfrac{1}{\sqrt{25-(x+\Delta x)}} - \dfrac{1}{\sqrt{25-x}}}{\Delta x}$$

$$\text{Aside: } \frac{\left(\sqrt{25-x} - \sqrt{25-(x+\Delta x)}\right)}{\left(\sqrt{25-(x+\Delta x)}\right)\left(\sqrt{25-x}\right)\Delta x} \cdot \frac{\left(\sqrt{25-x} + \sqrt{25-(x+\Delta x)}\right)}{\left(\sqrt{25-x} + \sqrt{25-(x+\Delta x)}\right)}$$

numerator

$$\cancel{25} - \cancel{x} - \cancel{25} + \cancel{x} + \Delta x = \Delta x$$

$$\lim_{\Delta x \to 0} \frac{\cancel{\Delta x}}{\left(\sqrt{25-(x+\Delta x)}\right)\left(\sqrt{25-x}\right)\cancel{\Delta x}\left(\sqrt{25-x} + \sqrt{25-(x+\Delta x)}\right)}$$

let $\Delta x = 0$

$$\boxed{f(x) = \frac{1}{2(25-x)\sqrt{25-x}}}$$

b) Use the definition to find the equation of the normal line to x = 9. (2 points)

$$f(9) = \frac{1}{\sqrt{25-9}} = \frac{1}{\sqrt{16}} = \frac{1}{4} \qquad \left(9, \tfrac{1}{4}\right)$$

point

$$f'(9) = \frac{1}{2(25-9)\sqrt{25-9}} = \frac{1}{2(16)(4)} = \frac{1}{128}$$

so slope for normal is -128

$$\boxed{y - \frac{1}{4} = -128(x-9)}$$