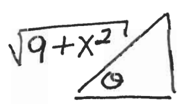
30 points each Quiz 2 & 3 PLEDGED Key

You have done your own work without the help of another person. You have used no outside sources except the reference sheet for this course. You have not observed another student seeking or giving help. You have completed this quiz in one sitting.

Beginning (time and date)__________ Ending (time and date)__________

Quiz 2: Evaluate the following. Show all work clearly on this sheet. (6 points each, 30 points)

a) $\int \frac{3}{(9+x^2)^2}dx$



$\theta = \text{Arctan}\left(\frac{x}{3}\right)$

$\tan\theta = \frac{x}{3}$

$x = 3\tan\theta$

$dx = 3\sec^2\theta\, d\theta$

$$\int \frac{3 \cdot 3\sec^2\theta\, d\theta}{(9+9\tan^2\theta)^2} = \int \frac{9\sec^2\theta\, d\theta}{\underbrace{(9\sec^2\theta)^2}_{81\sec^4\theta}}$$

$$\frac{1}{9}\int \frac{d\theta}{\sec^2\theta} = \frac{1}{9}\int \cos^2\theta\, d\theta$$

$$\frac{1}{9}\cdot\frac{1}{2}\int (1+\cos 2\theta)\, d\theta = \frac{1}{18}\theta + \frac{1}{36}\sin(2\theta) + C$$

$$\frac{1}{18}\theta + \frac{1}{18}\sin\theta\cos\theta + C$$

$$\frac{1}{18}\text{Arctan}\left(\frac{x}{3}\right) + \frac{1}{\cancel{18}_{\,6}}\left(\frac{x}{\sqrt{9+x^2}}\right)\left(\frac{\cancel{3}}{\sqrt{9+x^2}}\right) + C$$

$$\frac{1}{18}\text{Arctan}\left(\frac{x}{3}\right) + \frac{x}{6(9+x^2)} + C$$

b) $\int x\ln(x^2-4)dx$

$u = \ln(x^2-4)$ $dv = x\,dx$

$du = \frac{2x}{x^2-4}dx$ $v = \frac{1}{2}x^2$

$$\frac{1}{2}x^2\ln(x^2-4) - \int \frac{x^3}{x^2-4}dx$$

$u = x^2-4$, $x^2 = u+4$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$-\frac{1}{2}\int \frac{u+4}{u}du$

$-\frac{1}{2}\int 1 + \frac{4}{u}\,du$

$-\frac{1}{2}u - 2\ln|u| + C$

$$\frac{1}{2}x^2\ln(x^2-4) - \frac{1}{2}(x^2-4) - 2\ln|x^2-4| + C$$

c) $\int \frac{\sqrt{x+2}}{x-2}dx = \int \frac{\sqrt{x+2}}{x-2} \cdot \frac{\sqrt{x+2}}{\sqrt{x+2}} dx = 2\int \frac{u^2}{u^2-4} du$

$u = \sqrt{x+2}$

$du = \frac{1}{2\sqrt{x+2}} dx$

$2du = \frac{1}{\sqrt{x+2}}$

$u^2 = x+2$

$x = u^2 - 2$

$$2\int \frac{u^2-4+4}{u^2-4} du = 2\int \frac{\cancel{u^2-4}}{\cancel{u^2-4}} du + 8\int \frac{du}{u^2-4}$$

$\underline{\underline{2u}}$

$$\frac{8}{(u+1)(u-1)} = \frac{A}{u+1} + \frac{B}{u-1}$$

$$Au - A + Bu + B = 8$$

$\cancel{A}+B=0$

$-\cancel{A}+B=8$

$2B=4$

$B=2$

$A=-2$

$$\int \frac{-2}{u+1} du + \int \frac{2}{u-1} du$$

$$-2\ln|u+1| + 2\ln|u-1| + C$$

$$\boxed{2\sqrt{x+2} - 2\ln\left|\sqrt{x+2}+1\right| + 2\ln\left|\sqrt{x+2}-1\right| + C}$$

d) $\int \csc^3(2x)dx$

$u = \csc(2x) \quad dv = \csc^2(2x)dx$

$du = -2\csc(2x)\cot(2x) \quad v = -\frac{\cot(2x)}{2}$

$$\int \csc^3(2x)dx = -\frac{1}{2}\csc(2x)\cot(2x) - \int \csc(2x)\cot^2(2x)dx$$

$$-\int \csc(2x)[\csc^2(2x)-1]dx$$

$$\int \csc^3(2x)dx = -\frac{1}{2}\csc(2x)\cot(2x) - \int \csc^3(2x)dx + \int \csc(2x)dx$$

$$2\int \csc^3(2x)dx = -\frac{1}{2}\csc(2x)\cot(2x) + \frac{1}{2}\ln|\csc(2x) - \cot(2x)| + C$$

$$\int \csc^3(2x)dx = \boxed{-\frac{1}{4}\csc(2x)\cot(2x) + \frac{1}{4}\ln|\csc(2x)-\cot(2x)| + C}$$

e) $\int \frac{x\,dx}{\sqrt{-x^2+4x+5}} = \int \frac{x\,dx}{\sqrt{9-(x-2)^2}}$ $\int \frac{u+2}{\sqrt{9-u^2}}\,du$

$5+4 \quad -(x^2-4x+4)$

$9-(x-2)^2$

Complete the square

$u = x-2$

$du = dx$

$x = u+2$

$\int \frac{u}{\sqrt{9-u^2}}\,du + 2\int \frac{du}{\sqrt{9-u^2}}$

$w = 9-u^2$

$dw = -2u\,du$

$-\frac{1}{2}dw = u\,du$

$-\frac{1}{2}\int w^{-\frac{1}{2}}\,dw$

$-w^{\frac{1}{2}} + C$

$2\text{Arcsin}\left(\frac{u}{3}\right) + C$

$$\boxed{-\sqrt{9-(x-2)^2} + 2\text{Arcsin}\left(\frac{x-2}{3}\right) + C}$$

Quiz 3 Sketch the graphs of the following on the last page. Clearly label all aspects.

1. $y = \frac{\ln x^2}{\sqrt{x}}$ Answer the following. Write NONE if appropriate.

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Domain $x > 0$ 1 Asymptote(s) $x=0 \quad y=0$ 2 Intercept(s) $(1, 0)$ 1

Point(s) where the tangent line is horizontal $\left(e^2, \frac{4}{e}\right)$ 1 y' = $(4-\ln x^2)/2x^{3/2}$ 2

Points of inflection $\left(e^{\frac{8}{3}}, \frac{16}{3e^{4/3}}\right)$ 1 y'' = $-8+3\ln x/2x^{5/2}$ 2

2. $y = \frac{e^x}{x-1}$ Answer the following. Write NONE if appropriate.

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Domain $x \neq 1$ 1 Asymptote(s) $y=0 \quad x=1$ 2 Intercept(s) $(0, -1)$ 1

Point(s) where the tangent line is horizontal $(2, e^2)$ 1 y' = $e^x(x-2)/(x-1)^2$ 2

Point(s) of inflection NONE 1 y'' = $e^x(x^2-4x+5)/(x-1)^3$ 2

1.) $y = \dfrac{\ln x^2}{\sqrt{x}}$ $\quad D: x > 0$

$$\lim_{x \to 0^+} \frac{\ln x^2}{\sqrt{x}} = -\infty \quad \text{"}\frac{-\infty}{+0}\text{"} \qquad \boxed{x = 0 \ \text{V.A}}$$

$$\lim_{x \to +\infty} \frac{\ln x^2}{\sqrt{x}} = 0 \qquad \boxed{y = 0 \ \text{H.A.}}$$

$\sqrt{x}$ ← gets bigger faster

$$y = 0 = \frac{\ln x^2}{\sqrt{x}} \to \ln x^2 = 0 \to e^0 = x^2 \to x^2 = 1$$

$x = \pm 1$, $x = +1$ only (see Domain)

$\boxed{(1, 0) \text{ intercept}}$

$$y' = \frac{\sqrt{x} \cdot \frac{2}{x} - \ln x^2 \cdot \frac{1}{2\sqrt{x}}}{x} = \frac{\frac{2}{\sqrt{x}} - \frac{\ln x^2}{2\sqrt{x}}}{x} = \frac{4 - \ln x^2}{2x\sqrt{x}}$$

$$y' = \frac{4 - \ln x^2}{2x^{3/2}}$$

$$y' = \frac{\cancel{2}(2 - \ln x)}{\cancel{2}x^{3/2}}$$

$4 - \ln x^2 = 0$
$\ln x^2 = 4$
$e^4 = x^2$
$x = \pm e^2$
$x = +e^2$ only

$\left(e^2, \dfrac{\ln e^4}{e} = \dfrac{4}{e}\right)$
H.T.

$$y'' = \frac{x^{3/2}\left(-\frac{1}{x}\right) - (2 - \ln x) \cdot \frac{3}{2} x^{\frac{1}{2}}}{x^3} = \frac{-2x^{1/2} - 6x^{1/2} + 3x^{1/2} \ln x}{2x^3}$$

$$= \frac{-x^{1/2} - \frac{3(2 - \ln x)\, x^{1/2}}{2}}{x^3}$$

$$\frac{-\cancel{x^{1/2}}(8 - 3\ln x)}{2\cancel{x^3}\, x^{5/2}} = \frac{-(8 - 3\ln x)}{2x^{5/2}}$$

$$y'' = \frac{-8 + 3\ln x}{2x^{5/2}}$$

$-8 + 3\ln x = 0$
$\ln x = \dfrac{8}{3}$
$e^{\frac{8}{3}} = x$

$\left(e^{\frac{8}{3}}, \dfrac{16}{3e^{4/3}}\right)$
P.I.

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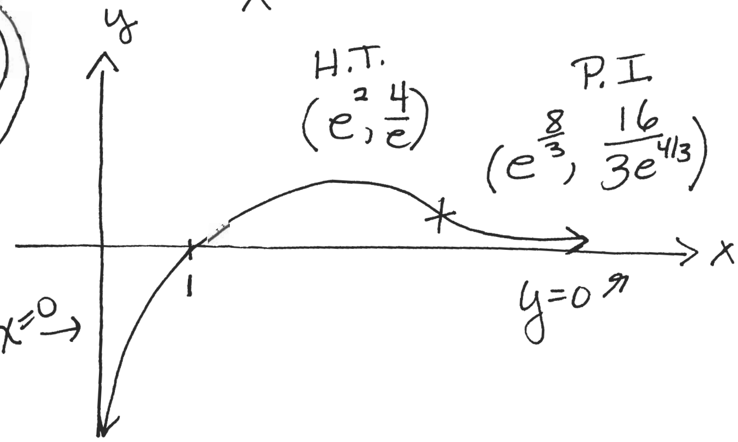


2) $y = \dfrac{e^x}{x-1}$ D: $x \neq 1$

$x=0,\ y = \dfrac{e^0}{-1} = -1$ $\boxed{(0,-1) \text{ intercept}}$

$\lim\limits_{x\to+\infty} \dfrac{e^x}{x-1} = +\infty$ upward (e^x ← bigger faster)

$\lim\limits_{x\to-\infty} \dfrac{e^x}{x-1} = 0$ $\boxed{y=0 \text{ H.A. on left}}$ (e^x ← zero; $x-1$ ← $-\infty$)

$y' = \dfrac{(x-1)e^x - e^x(1)}{(x-1)^2} = \dfrac{xe^x - e^x - e^x}{(x-1)^2}$

$y' = \dfrac{e^x(x-2)}{(x-1)^2}$ $\boxed{x=2 \text{ H.T. } (2,e^2)}$

$y'' = \dfrac{(x-1)^2\left[e^x(1) + (x-2)e^x\right] - e^x(x-2)2(x-1)}{(x-1)^4}$

$y'' = \dfrac{e^x\cancel{(x-1)}\left[(x-1)^2 - 2(x-2)\right]}{(x-1)^{\cancel{4}3}} = \dfrac{e^x(x^2-2x+1-2x+4)}{(x-1)^3}$

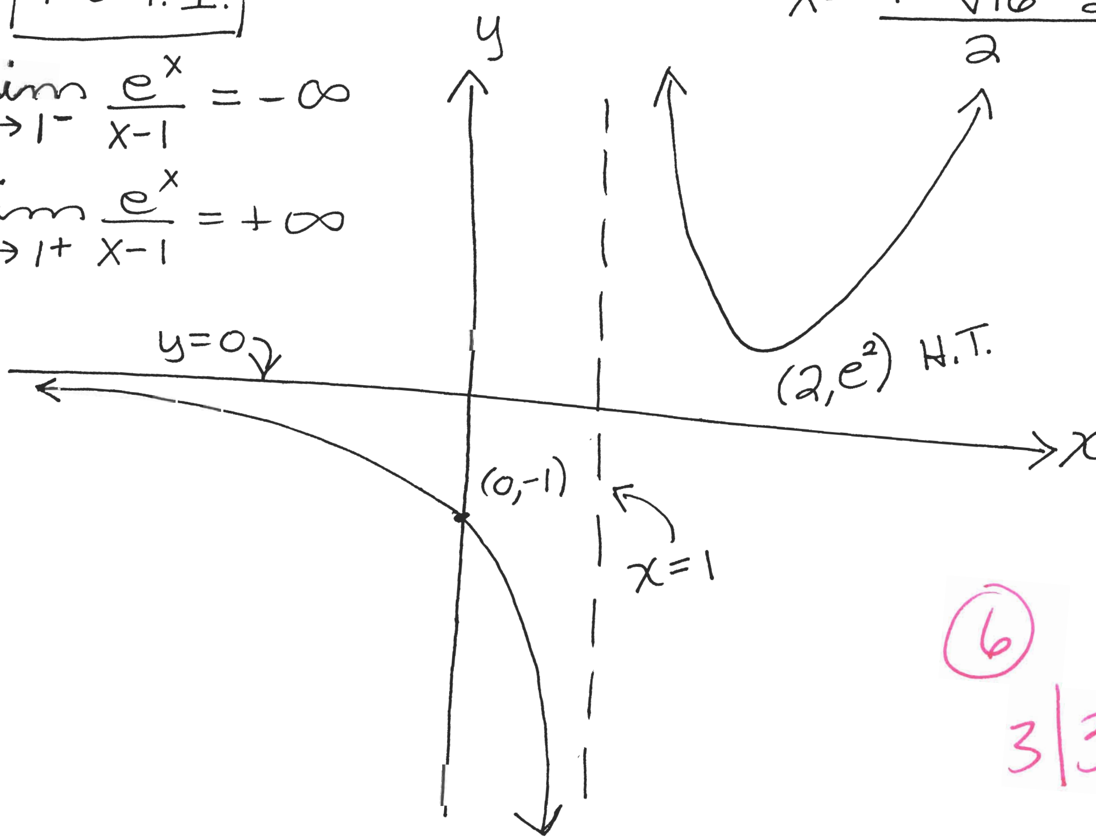
$y'' = \dfrac{e^x(x^2-4x+5)}{(x-1)^3} \Rightarrow x^2-4x+5=0$

$x = \dfrac{4 \pm \sqrt{16-20}}{2}$ $\varnothing$

$\boxed{\text{No P.I.}}$

$\lim\limits_{x\to1^-} \dfrac{e^x}{x-1} = -\infty$

$\lim\limits_{x\to1^+} \dfrac{e^x}{x-1} = +\infty$



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