Math 107 – Test 3
March 20, 2009
(108 points)

Pledged Key

Instructions: Show all work and the set-ups for all calculations.

10 pts **1.** The data below are the weights (in grams) for a sample of plain M&M's. Find a 95% confidence interval for the true mean weight of plain M&M's. Assume that the weights are normally distributed.

$\bar{x} = .8635$
$s = .0576$
$n = 13$

.751	.841	.856	.799	.966	.859	.857
.942	.873	.809	.890	.878	.905	

Check the assumptions. $n < 30$, σ unknown, approx. normal

Show work:

$$E = t_{\alpha/2}\, s/\sqrt{n}$$
$$= 2.179\,(.0576/\sqrt{13})$$
$$= .0348$$
$$(\bar{x} - E, \bar{x} + E) = (.8287, .8983)$$

Answer with a complete sentence:

We are 95% confident the mean weight of plain M+M's is between .8287 and .8983 grams.

8 pts **2.** How large a sample is needed to determine the mean weight of plain M&M's with 95% confidence within .01 grams? Use the standard deviation from the sample in problem 1.

$$n = \left(\frac{z_{\alpha/2}\,\sigma}{E}\right)^2 = \left(\frac{1.96\,(.0576)}{.01}\right)^2 = 127.45 < 128$$

20 pts **3.** A sample of 40 plain M&M's has mean weight .848 grams and standard deviation .051 grams. Use a significance level of .05 to test the claim that the mean weight of plain M&M's is less than .860 grams.

Check the assumptions. $n \geq 30$

Null Hypothesis $\mu \geq .86$

Alternative Hypothesis $\mu < .86$

Critical Value -1.645

Test Value -1.49

Show the set-up for your calculations and the graph for the critical value.

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{.848 - .860}{.051/\sqrt{40}} = -1.49$$

Decision and reason: Fail to reject H_0 because the test value is not in the rejection region.

Summary: The mean weight is not significantly less than .86 grams.

There is not enough evidence to support the claim that the mean weight is less than .86 grams.

8 pts **4.** Error types.

(a) Describe the two types of error of hypothesis testing.

Type I : reject a true H_0

Type II : fail to reject a false H_0.

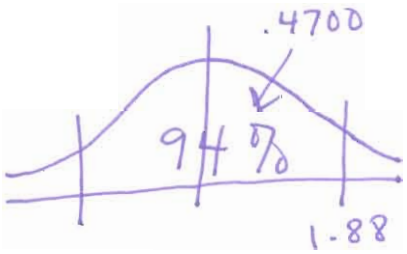
(b) What type of error is your result from problem 3 vulnerable to and why?

Since we failed to reject H_0, we could have made a type II error.

10 pts **5.** A sample of 100 M&M's contains 12 small M&M's. ("Small" means less than .8 grams in weight.) Find a 94% confidence interval for the proportion of small M&M's.

Check the assumptions. $n\hat{p} = 12 \geq 5$ $n\hat{q} = 88 \geq 5$

Show work: $E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.88\sqrt{\frac{(.12)(.88)}{100}} \approx .061$



$(\hat{p} - E, \hat{p} + E)$

$= (.059, .181)$

Answer with a complete sentence:

We are 94% confident the proportion of small M+M's is between 5.9% and 18.1%.

30 pts **6.** Assume that the weights of plain M&M's are normally distributed with a population mean of .860 grams and standard deviation of .050 grams.

(a) Find the probability that a randomly selected plain M&M's weight is between .9 and 1.0 grams.

$P(.9 < x < 1.0)$

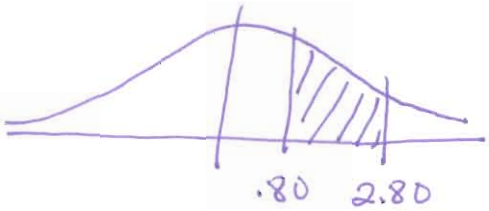
$= P(.80 < z < 2.80)$

$= .4974 - .2881$

$= .2093$

$z = \frac{.9 - .86}{.05} = .80$

$z = \frac{1.0 - .86}{.05} = 2.80$



(b) Find the probability that a sample of 10 plain M&M's has mean weight between .8 and .9 grams.

$P(.8 < \bar{x} < .9)$

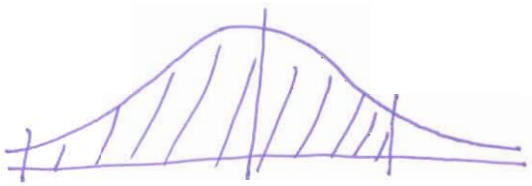
$= P(-3.79 < z < 2.53)$

$= .4999 + .4943$

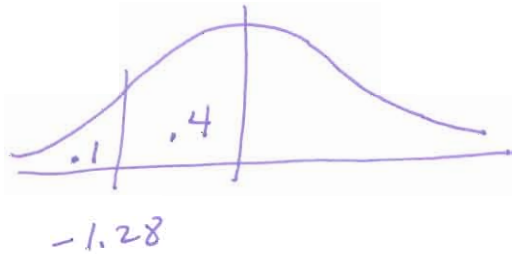
$= .9942$

$z = \frac{.8 - .86}{.05/\sqrt{10}} = -3.79$

$z = \frac{.9 - .86}{.05/\sqrt{10}} = 2.53$



(c) Find the 10th percentile weight of plain M&M's.



$-1.28 = \frac{x - .860}{.05}$

$x = .796$ grams

12 pts **7.** 3.5% of plain M&M's are large. ("Large" means greater than .95 grams in weight.) Find the probability that a sample of 200 plain M&M's contains more than 10 large M&M's.

Check the assumptions. $np = 7 \geq 5$ $nq = 193 \geq 5$

$P(x > 10)$

(Continuity correction)

$= P(x > 10.5)$

$= P(z > 1.35)$

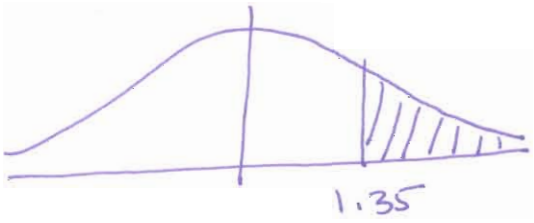
$= .5 - .4115$

$= .0885$

$\mu = 200(.035) = 7$

$\sigma = \sqrt{200(.035)(.965)} \approx 2.60$

$z = \frac{10.5 - 7}{2.6} \approx 1.35$



10 pts **8.** Match each of the descriptions below with one of the following names:
Archimedes, Democritus, Euclid, Fermat, Fibonacci, Kepler, Napier, Pascal, Pythagoras, Thales.

Fibonacci Described a pattern of numbers (1, 1, 2, 3, 5, ...) that often arises in mathematical biology.

Napier A Scottish laird who invented logarithms.

Pascal A child prodigy who wrote an essay on conic sections at age 16.

Pythagoras Taught that there is a basic relationship between musical harmony and mathematics.

Archimedes After determining the distances of the planets from the earth, constructed a spherical planetarium imitating the motion of the sun, moon and six then known planets.