

Name _____

106 Points

3/4/11

(pledged)

Show work for credit.

Solve the following. Check where appropriate. (6 pts. each)

1. $d^{2/3} + d^{1/3} - 6 = 0$

$$(d^{1/3} + 3)(d^{1/3} - 2) = 0$$

$$d^{1/3} + 3 = 0$$

$$d^{1/3} = -3$$

$$(d^{1/3})^3 = (-3)^3$$

$$d = -27$$

$$d^{1/3} - 2 = 0$$

$$d^{1/3} = 2$$

$$(d^{1/3})^3 = 2^3$$

$$d = 8$$

$$d = 8, -27$$

2. $\frac{2x+2}{x^2-4x+3} + \frac{6x}{x-3} = \frac{3x}{x-1}$

$$\frac{2x+2}{(x-3)(x-1)} + \frac{6x}{x-3} = \frac{3x}{x-1}$$

$$2x+2 + 6x^2 - 6x = 3x^2 - 9x$$

$$3x^2 + 5x + 2 = 0$$

$$(3x+2)(x+1) = 0$$

$$3x+2=0 \quad | \quad x+1=0$$

$$3x = -2$$

$$x = -\frac{2}{3} \checkmark$$

$$x+1=0$$

$$x = -1 \checkmark$$

(check)

$$x = -\frac{2}{3}, -1 \checkmark$$

3. Use an appropriate substitution of variable to solve $3\left(\frac{1}{x}+3\right)^2 - 5\left(\frac{1}{x}+3\right) - 2 = 0$ (6)

let $u = \frac{1}{x} + 3$

$$3u^2 - 5u - 2 = 0$$

$$(3u+1)(u-2) = 0$$

$$3u+1=0 \quad | \quad u-2=0$$

$$3u=-1 \quad | \quad u=2$$

$$u = -\frac{1}{3}$$

$$\frac{1}{x} + 3 = -\frac{1}{3}$$

$$\frac{1}{x} = -\frac{1}{3} - \frac{9}{3} = -\frac{10}{3}$$

so $x = -\frac{3}{10}$ ✓

$$\frac{1}{x} + 3 = 2$$

$$\frac{1}{x} = -1$$

so $x = -1$ ✓

$x = -1, -\frac{3}{10}$ ✓

4. $3\left[\frac{1}{3} - 4\left(x + \frac{2}{3}\right)\right] > -2\left(\frac{1}{2} - x\right)$ (6)

$$3\left[\frac{1}{3} - 4x - \frac{8}{3}\right] > -1 + 2x$$

$$1 - 12x - 8 > -1 + 2x$$

$$-12x - 7 > -1 + 2x$$

$$-14x > 6$$

$$x < -\frac{6}{14}$$

$x < -\frac{3}{7}$

5. If $f(x) = x^2 - 5x + 1$, find $\frac{f(a+h) - f(a)}{h}$ (6)

$$\frac{f(a+h) - f(a)}{h} = \frac{(a+h)^2 - 5(a+h) + 1 - (a^2 - 5a + 1)}{h}$$

$$= \frac{\cancel{a^2} + 2ah + h^2 - 5a - 5h + 1 - (\cancel{a^2} - 5a + 1)}{h} = \frac{2ah + h^2 - 5h}{h}$$

$= 2a + h - 5$

6. If $f(x) = 5x - 3$ and $g(x) = \frac{4}{x-2}$, find

(3 each)

a) $(f+g)(-3)$

$$= 5x - 3 + \frac{4}{x-2} = 5(-3) - 3 + \frac{4}{-3-2} = -15 - 3 - \frac{4}{5} = -18\frac{4}{5}$$

or $-\frac{94}{5}$

b) $\left(\frac{f}{g}\right)(x) = \frac{5x-3}{\frac{4}{x-2}} = (5x-3)\left(\frac{x-2}{4}\right) = \frac{5x^2 - 13x + 6}{4}$ or $\frac{5}{4}x^2 - \frac{13}{4}x + \frac{3}{2}$

c) The domain in part b $\mathbb{R}, x \neq 2$

d) $(g \circ f)(x) = g(f(x)) = g(5x-3)$

$$= \frac{4}{(5x-3)-2} = \frac{4}{5x-3-2}$$

$$= \frac{4}{5x-5}$$

(8 each)

7. If $f(x) = 5x - 1$, a) find f^{-1} b) verify your findings

$$f(x) = 5x - 1$$

$$y = 5x - 1$$

swap

$$x = 5y - 1$$

$$5y = x + 1$$

$$y = \frac{1}{5}x + \frac{1}{5}$$

$$f^{-1}(x) = \frac{1}{5}x + \frac{1}{5}$$

verify:

$$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\frac{1}{5}x + \frac{1}{5}\right)$$

$$= 5\left(\frac{1}{5}x + \frac{1}{5}\right) - 1 = x + 1 - 1 = x \checkmark$$

$$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(5x - 1)$$

$$= \frac{1}{5}(5x - 1) + \frac{1}{5}$$

$$= x - \frac{1}{5} + \frac{1}{5}$$

$$= x \checkmark$$

