# Math 111
## Review Handout for Final

[Answers begin on page 8]

1.) a) State the definition of derivative.

b) Use the definition of derivative to find $f'(x)$ for: ① $f(x) = \frac{1}{\sqrt{x-2}}$ ② $f(x) = \frac{x}{2x-1}$

2.) Let $f(x) = \begin{cases} \frac{4}{x+4} & \text{if } x < -3 \\ x^2 - 5 & \text{if } -3 < x < 0 \\ \sqrt{25-x^2} & \text{if } x \geq 0 \end{cases}$

Discuss any discontinuities.

3.) Evaluate:

a) $\lim_{x \to 7} \frac{3 - \sqrt{x+2}}{x-7}$

b) $\lim_{x \to +\infty} \frac{x^2 - 3x^3}{x^3 + 2x - 1}$

c) $\lim_{x \to -1^-} \frac{|x+1|}{x^2-1}$

d) $\lim_{x \to 1^-} \frac{|x+1|}{x^2-1}$

e) $\lim_{x \to -\infty} \frac{x^2 - 4x^5}{1 - x^4}$

f) $\lim_{x \to +\infty} \frac{3x^2}{(2x+1)^2}$

4.) Set up integrals using the disk and shell methods to find the volume of the solid formed when the region bounded by $y = x^2 + 1$ and $y = 9 - x^2$ is revolved about a) $y = -1$ and b) $x = -3$.

5.) Find the equation of the normal line to $y = \cot\left(\frac{\pi}{4} - x\right)$ at $x = 0$.

6.) Does the Mean Value Theorem apply to $f(x) = \sqrt{3x+1}$ on the interval $[1,5]$? [Mean Value Theorem for Derivatives.]

7.) Evaluate:

a) $\int \frac{x+2}{\sqrt{-x^2-4x}}\,dx$ b) $\int \frac{1}{x^2+4x+4}\,dx$ c) $\int \frac{3e^{2x}}{\sqrt{e^x+1}}\,dx$

8.) Sketch the graph of each of the following. Label all important points.

*a) $y = x^{\frac{1}{3}}(3-x)^{\frac{2}{3}}$ *b) $y = \frac{x+2}{x^2-x-2}$ c) $y = 3x^4 - 4x^3 + 1$

d) $y = \frac{2x^3}{x^3+1}$ * Do not check concavity on a & b

9.) Find $y'''$ for $y = \sqrt{3-x^2}$.

10.) Let $f(x) = \frac{x^3+x-6}{x^3-2x^2}$. Find:

a) $\lim_{x \to 2^+} f(x)$ b) $\lim_{x \to 0^-} f(x)$ c) domain d) asymptotes

e) intercepts f) $f'''(x)$

11.) Find $f(x)$ if $f''(x)=12$, $f(1)=2$ and $f(2)=15$.

12.) Evaluate:

a) $\int \frac{(x-2)^2}{\sqrt{x}}\,dx$ b) $\int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{\cos^4 x}\,dx$ c) $\int \frac{dx}{\sec^3 x}$

d) $\int \frac{4\sec^2 5x}{1-3\tan 4x}\,dx$ e) $\int \tan^2(2x)\,dx$

13.) Using calculus, show that the volume of a cylinder is $V=\pi r^2 h$.

14.) Find the average value for $f(x)=\cos(2x)$ over $[0, \frac{\pi}{2}]$. Does the Mean Value Theorem for Integrals apply?

15.) Evaluate:

a) $\lim_{x\to 2^-} \frac{|x-2|}{x^2-4}$ b) $\lim_{x\to+\infty} \frac{x-x^2}{3x^2+1}$ c) $\lim_{x\to-\infty} \frac{x^3+3x^2}{x^2-4}$

d) $\lim_{x\to 0^+} \cot x$ e) $\lim_{x\to 4^-} \frac{x}{x-4}$ f) $\lim_{x\to -3^-} \frac{x^2}{x^2-9}$

g) $\lim_{x\to-\infty} \frac{2x^2-3}{1-x^2}$ h) $\lim_{x\to -1^-} \frac{x}{x+1}$ i) $\lim_{x\to\infty} \frac{4x^2-2x+1}{x^2-4}$

j) $\lim_{x\to-\infty} \frac{x^3-3x^2}{x^2-2}$ k) $\lim_{x\to 0^+} \frac{1}{x}$ l) $\lim_{x\to 2^-} \frac{1}{x^2-4}$

m) $\lim_{x\to 2^+} \frac{x^3-8}{x^2-4}$ n) $\lim_{x\to 2^-} \frac{|x+2|}{x^2-4}$ o) $\lim_{x\to \frac{\pi}{2}^-} \tan x$

p) $\lim_{x\to \pi^+} \csc x$ q) $\lim_{x\to+\infty} \frac{x^2-2x+4}{x-2}$ r) $\lim_{x\to 2^-} \frac{x^2-2x+4}{x-2}$

16.) A four foot tall boy drops a ball from his height above a bridge. The ball hits the water 3 seconds later. How high is the bridge above the water?

17.) Differentiate and simplify:
a) $f(x) = \sqrt{\ln(\sec 4x)}$ b) $y = 2x^2e^{2x} - 2xe^{2x} + e^{2x}$

18.) Under what three general conditions will a derivative not exist?

19.) For a function to be continuous at a point $(x=a)$, then f must meet three conditions. What are they?

20.) Evaluate:
a) $\int e^{\cos^2 x} \sin x \cos x \, dx$ b) $\int \frac{dx}{e^{1/x^2} x^3}$

21.) Solve: $\frac{dy}{dx} = \frac{\sin(3x)\sin^2(2y)}{\cos^3(3x)\cot(2y)}$

22.) Differentiate, simplify where possible:
a) $y = \frac{\sqrt{x}\sin(4x)}{\cos(4x)}$ b) $y = \sin^2 x^2 - \cos^2 x^2$
c) $y = e^{2\ln x}$

23.) Does the Mean Value Theorem for Derivatives apply to $f(x) = x^2 - 4x$ on the interval $[2,6]$?

24.) A ball is thrown vertically upward from a window with an initial velocity of 48 ft/sec. If the window is 64 feet above the ground, find the following:
a) maximum height reached by the ball.
b) the time required to reach the ground.
c) the velocity at the time of impact.

25.) Prove: If $y = x^{\frac{p}{q}}$, then $y' = \frac{p}{q} x^{\frac{p}{q}-1}$

26.) Differentiate: $2(xy)^{3/2} + y^2 = x^2$

27.) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for: a) $y^3 = x^2 - 17$, b) $x^2y^3 = 1$

28.) Find an equation for the tangent line to the curve $x^3 - x + xy + y^3 + 24 = 0$ at $(-1, -3)$.

29.) Sketch the graphs, indicate the region, and set up integrals both ways to find the area enclosed by $x = y^2 - 2y$ and $x = -y^2 + 4$. Work one way to completion.

30.) a) $\int x\sqrt{x+1}\,dx$ b) $\int \frac{3x^5 - x^3}{\sqrt{4-x^2}}\,dx$ c) $\int (1-\sqrt{x})^2\,dx$

d) $\int \frac{2x+4}{\sqrt{x^2+4x+4}}\,dx$ e) $\int \cos^3\left(\frac{x}{2}\right)dx$ f) $\int \sqrt{x^4+4x^2+4}\,dx$

g) $\int \frac{6x^2}{\sqrt[3]{x^3+5}}\,dx$ h) $\int \frac{\sin\left(\frac{1}{x^2}\right)}{x^3}\,dx$ i) $\int \frac{x^2+x}{(2x^3+3x^2+1)^3}\,dx$

j) $\int \cot^2\left(\frac{x}{2}\right)dx$ k) $\int \frac{(x+4)^2}{\sqrt{x}}\,dx$ l) $\int \left(\frac{1}{x^2} + \frac{1}{\sqrt{x}}\right)dx$

31.) Differentiate and simplify:

a) $y = \cos(\sin 2x)$ b) $y = x\cos(\sin x)$ c) $y = \left(\frac{x^2+1}{x}\right)^4$

32.) A ladder 20 feet long leans against a vertical building. If the bottom of the ladder slides away from the building horizontally at a rate of 2 ft/sec, how fast is the ladder sliding down the building when the top of the ladder is 12 feet about the ground?

33.) An open box is formed from a rectangular sheet of cardboard by cutting equal squares from each corner and folding up the edges. If the dimensions of the cardboard are 15 in x 24 in, what size squares should be cut to obtain a box of maximum volume?

34.) If $x$ is the number of items produced in one week, the selling price per item is $\$(100-0.02x)$. The total cost for $x$ items per week is $\$(40x+1500)$. How many items should be produced and sold per week for maximum profit? What is the maximum profit?

35.) Sketch the graphs of:

a) $f(x) = x^3 - 6x^2 + 9x + 3$ (omit x-intercept(s))

b) $f(x) = \frac{8x}{x^2+4}$ c) $f(x) = x(1-x)^{2/5}$

36.) A conical tank has radius 3 ft and depth 10 ft. If water is poured into the tank at the rate of $2\pi$ ft$^3$/min, how fast is the water level rising when the water in the tank is 5 ft deep?
[$V = \frac{1}{3}\pi r^2 h$ for cone]

37.) A ball is thrown upward from the top of a building with a velocity 60 ft/sec. Four seconds later the ball strikes the ground at the base of building. Find: a) the height of the building; b) the maximum height reached by the ball; c) the velocity of the ball when it strikes the ground.

38.) Prove by induction: $\sum_{k=1}^{n}(k^3-k) = \frac{n^4+2n^3-n^2-2n}{4} = \frac{n(n^2-1)(n+2)}{4}$

39.) Evaluate:

a) $\int_0^{\frac{\pi}{4}} \sin^2 2x \cos 2x \, dx$

b) $\int_1^4 \frac{dx}{\sqrt{x}(4-\sqrt{x})^3}$

c) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 9\cos 2x \, dx$

d) $\int_0^{\frac{\pi}{8}} \sec^2 2x \, dx$

1.) a) $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$, if limit exists

b) ① $f'(x) = \frac{-1}{2(x-2)\sqrt{x-2}}$ ② $f'(x) = \frac{-1}{(2x-1)^2}$ must show all steps clearly.

2.) $x = -4$ is not in the domain; $x = -3$ is not in the domain; $x = 0$ since $\lim_{x \to 0} f(x)$ DNE, $\lim_{x \to 0^+} f(x) = 5 \neq \lim_{x \to 0^-} = -5$

3.) a) $-\frac{1}{6}$ b) $-3$ c) $\frac{1}{2}$ d) $-\infty$ e) $-\infty$ f) $\frac{3}{4}$

4.) a)

D: $\pi \int_{-2}^{2} \left\{ \left[ (9-x^2) + 1 \right]^2 - \left[ (x^2+1) + 1 \right]^2 \right\} dx$

S: $2\pi \int_{1}^{5} (y+1)(2\sqrt{y-1})\, dy + 2\pi \int_{5}^{9} (y+1)(2\sqrt{9-y})\, dy$

b)

D: $\pi \int_{1}^{5} \left\{ \left[ \sqrt{y-1} + 3 \right]^2 - \left[ -\sqrt{y-1} + 3 \right]^2 \right\} dy + \pi \int_{5}^{9} \left\{ \left[ \sqrt{9-y} + 3 \right]^2 - \left[ -\sqrt{9-y} + 3 \right]^2 \right\} dy$

S: $2\pi \int_{-2}^{2} (x+3)(8-2x^2)\, dx$

5.) $y - 1 = -\frac{1}{2}x$

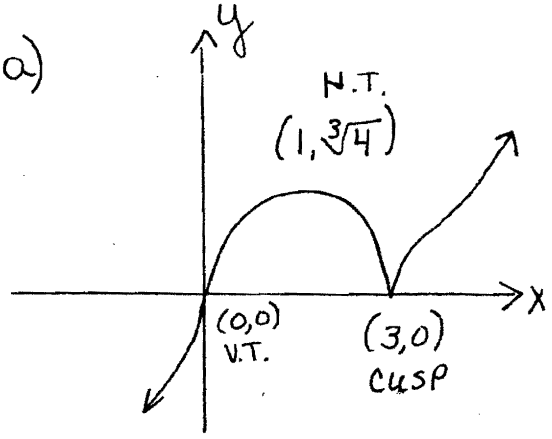
6.) yes, $c = \frac{8}{3}$ state assumptions clearly.

7.) a) $-\sqrt{-x^2-4x} + C$

b) $\frac{-1}{x+2} + C$

c) $2(e^x+1)^{\frac{3}{2}} - 6(e^x+1)^{\frac{1}{2}} + C$

8. a)

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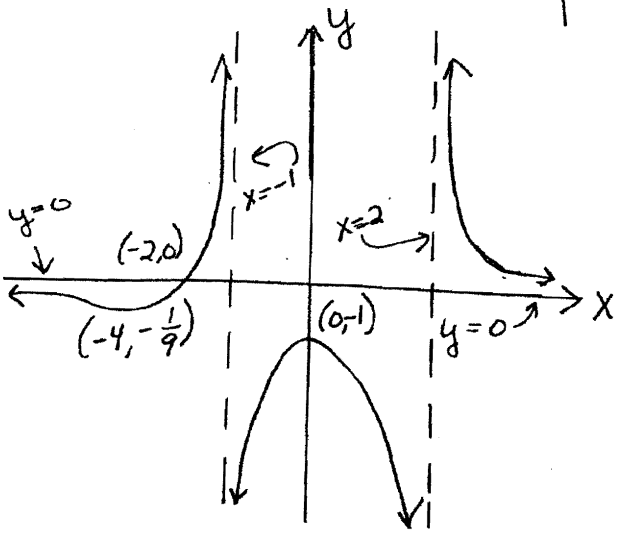


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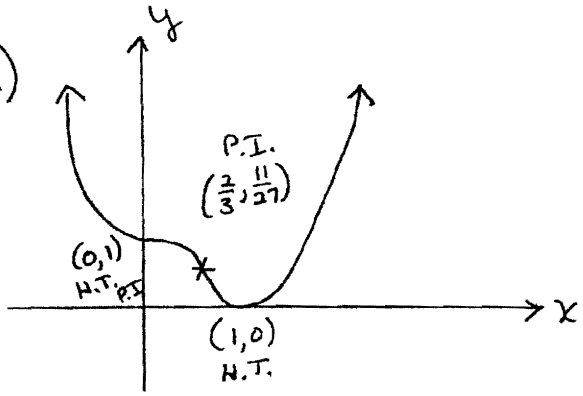


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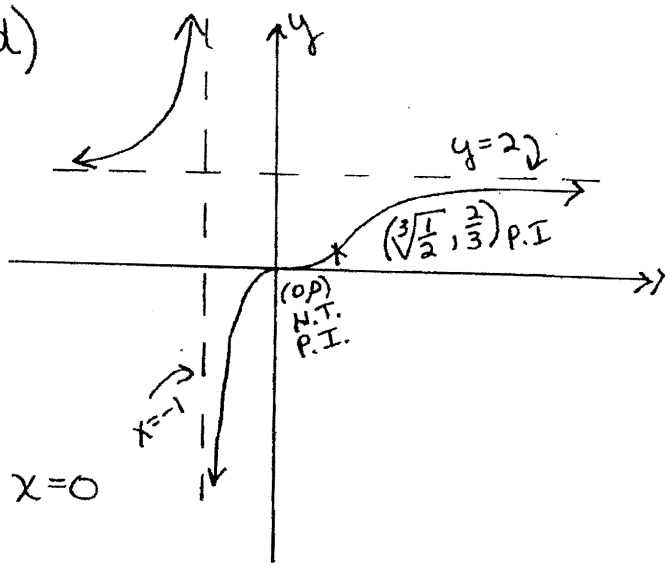



9.) $y''' = \dfrac{-9x}{(3-x^2)^{5/2}}$

10.) a) $\frac{5}{4}$ b) $+\infty$ c) $x \neq 0, 2$ d) $y=0, x=0$
e) $(-3,0)$ f) $f'''(x) = \dfrac{-6x-72}{x^5}$

11.) $f(x) = 6x^2 - 5x + 1$

12.) a) $\frac{2}{5}x^{5/2} - \frac{8}{3}x^{3/2} + 8x^{\frac{1}{2}} + C$ b) $\frac{1}{3}$ c) $\sin x - \frac{1}{3}\sin^3 x + C$
d) $-\frac{4}{15}\ln|1-3\tan 5x| + C$ e) $\frac{1}{2}\tan(2x) - x + C$

14.) 0; yes, f is continuous, $c = \frac{\pi}{4}$

15.) a) $-\frac{1}{4}$ b) $-\frac{1}{3}$ c) $-\infty$ d) $+\infty$ e) $-\infty$ f) $+\infty$ g) $-2$ h) $+\infty$
i) 4 j) $-\infty$ k) $+\infty$ l) $-\infty$ m) 3 n) $-\infty$ o) $+\infty$ p) $-\infty$
q) $+\infty$ r) $-\infty$ s) 2 t) $+\infty$ u) $+1$ v) $-1$ w) $+\infty$

16.) 140 ft $\left[h(t) = -16t^2 + 144\right]$

17.) no since $f'(x) = \frac{1}{3x^{2/3}}$ is undefined at $x = 0$.

18.) } See class notes
19.) }
20.) a) $-\frac{1}{2}e^{\cos^2 x} + C$ b) $\frac{1}{2}e^{-\frac{1}{x^2}} + C$

21.) $\frac{1}{6}\sec^2(3x) + \frac{1}{4}\cot^2(2y) = C$

22.) a) $y' = \frac{8x\sec^2(4x) + \tan(4x)}{2\sqrt{x}}$ b) $y' = 8x\sin x^2 \cos x^2$
c) $\frac{dy}{dx} = 2x$

23.) yes. f is continuous over $[2,6]$ } $2 < c = 4 < 6$
f' is continuous over $(2,6)$ }

24.) a) 100 ft b) 4 sec c) $-80$ ft/sec

25.) See class notes 26.) $\frac{dy}{dx} = \frac{2x - 3y\sqrt{xy}}{3x\sqrt{xy} + 2y}$

27.) a) $\frac{dy}{dx} = \frac{2x}{3y^2}$ $\frac{d^2y}{dx^2} = \frac{6y^3 - 8x^2}{9y^5}$ b) $\frac{dy}{dx} = -\frac{2y}{3x}$ $\frac{d^2y}{dx^2} = \frac{10y}{9x^2}$

28.) $y + 3 = \frac{1}{26}(x + 1)$

29.) $A = \int_{-1}^{2}\left[-y^2 + 4 - (y^2 - 2y)\right]dy = 9$

$A = \int_{-1}^{0}\left[1 + \sqrt{1+x} - (1 - \sqrt{1+x})\right]dy + \int_{0}^{3}\left[\sqrt{4-x} - (1 - \sqrt{1+x})\right]dy$

$+ \int_{3}^{4}\left[\sqrt{4-x} - (-\sqrt{4-x})\right]dy$

30.) a) $\frac{6}{15}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C$ b) $-44\sqrt{4-x^2} + \frac{23}{3}(4-x^2)^{3/2} + C$

c) $x - \frac{4}{3}x^{3/2} + \frac{1}{2}x^2 + C$

d) $2\sqrt{x^2+4x+4} + C$ e) $2\sin\left(\frac{x}{2}\right) - \frac{2}{3}\sin^3\left(\frac{x}{2}\right) + C$

f) $\frac{1}{3}x^3 + 2x + C$ g) $3(x^3+5)^{2/3} + C$ h) $\frac{1}{2}\cos\left(\frac{1}{x^2}\right) + C$

i) $\frac{-1}{12}(2x^3+3x^2+1)^{-2} + C$ j) $2\cot\left(\frac{x}{2}\right) - x + C$

k) $\frac{1}{2}x^2 + 8x + 16\ln|x| + C$ l) $-\frac{1}{x} + 2\sqrt{x} + C$

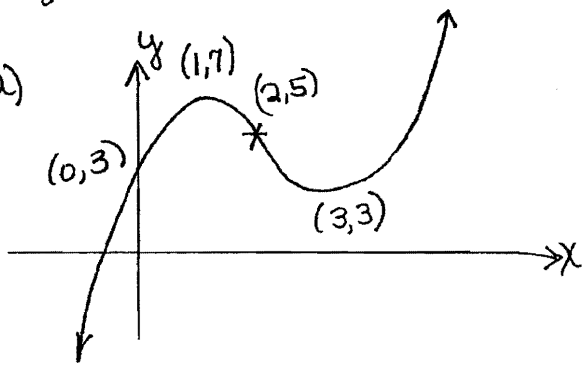
31.) a) $y' = -2\sin(\sin 2x)\cos 2x$

b) $y' = \cos(\sin x) - x\sin(\sin x)\cos x$

c) $y' = \frac{4(x^2+1)^3(x^2-1)}{x^5}$

32.) $\frac{8}{3}$ ft/sec 33.) 3 in x 3 in 34.) 1500 $30,000

35.) a)

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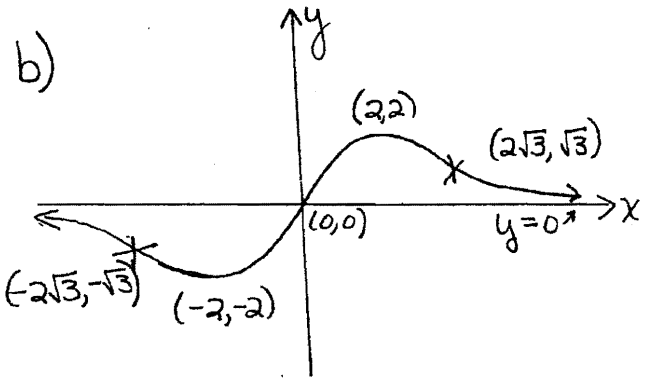


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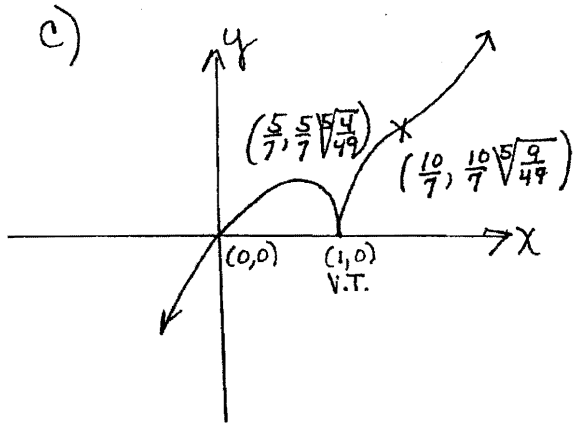


36.) $\frac{8}{9}$ ft/min

37.) a) 16 ft b) $72\frac{1}{4}$ ft c) -68 ft/sec

39.) a) $\frac{1}{6}$ b) $\frac{5}{36}$ c) 9 d) $\frac{1}{2}$