Proportions: Confidence Intervals and Hypothesis Testing

Definitions:

unbiased estimator	point estimate
confidence interval	maximum error of estimate or
Critical Value (CV)	margin of error (E)
alpha, beta	significance level and
hypothesis	p-value or probability value
testing hypotheses	test statistic
null hypothesis	Type I (α) and Type II (β) errors
significant majority	significantly different

Know:

* Be able to approximate a binomial distribution with the standard normal distribution
* Be able to find and to interpret confidence intervals for given proportions.
* Be able to determine the minimum sample size needed to estimate a proportion for a specific error value.
* Be able to show the hypothesis testing procedure to compare a proportion of a sample to a given value and to compare two proportions. Be able to relate the hypothesis testing procedure to the related confidence interval.

Normal Approximation to the Binomial

The normal distribution can approximate the binomial distribution IF $np \geq 5$ and $nq \geq 5$, with $\mu = np$ and $\delta = \sqrt{npq}$.

There needs to be a continuity correction factor of .5 to model the discrete distribution (binomial) with a continuous distribution (standard normal).

The binomial:

$P(x) = {}_nC_x p^x q^{n-x}$ with mean and variance of

$\mu = np$ and $\sigma^2 = npq$

Example:

Let X be the random variable that represents a count of the number of heads showing when a coin is tossed 12 times. The binomial probabilities can be found by using the following:

$P(x) = {}_{12}C_x (.5)^x (.5)^{12-x}$ for x = 0, 1, 2, . . . 12; Note that n = 12, p = .5, q = .5

Figure the following probabilities:

x	P(x)
0	.000244
1	.002929
2	.016111
3	.053702
4	.120830
5	.193327
6	.225548
7	.193327
8	.120830
9	.053702
10	.016111
11	.002929
12	.000244

$\mu = np = (12)(.5) = 6$

$\delta^2 = npq = 12(.5)(.5)$

δ is approximately 1.732

1. P(at least 8 heads)

As a binomial: P(8) + P(9) + P(10) + P(11) + P(12) = .1938 (add values above and round off)

As a normal approximation: Use 7.5 as the correction factor and find z:

$z = \frac{7.5 - 6}{1.732}$ is approximately .87.

And from the standard normal table: P(z >.87) = .1922

2. P(exactly 5 heads)

As a binomial: P(5) = .1933 (from the value in the table)

As a normal approximation:

P(4.5 < x < 5.5) = P(-.87 < z < -.29) = .1937

3. P(at most 5 heads)

As a binomial: P(0) + P(1) + P(2) + P(3) + P(4) + P(5) = .3871 (add values above, round off)

As a normal approximation: P(x < 5.5) = P(z < -.29) = .3859

Example Problems:

1. Toss a coin 16 times. Let X be the number of heads that appear. Find the probability that there will be more than 13 heads. (a) Work as a binomial. (b) Use the standard normal to approximate the probability. (c) Was the approximation reasonable? Explain.

2. Assume that the probability of a college student having a car on campus is .30. A random sample of 12 students is taken. What is the probability that at least 4 will have a car on campus? (a) Work the problem as a binomial. (b) Approximate the probability using the standard normal. (c) Is the approximation reasonable? Explain clearly.

Answers:

1. This is a binomial with n = 16 and p = 1/2.
(a) P(14) + P(15) + P(16) is approximately .0021.
(b) P(x > 13.5) = P(z > 2.75) = .003 (the area under the standard normal distribution from z = 2.75 to the right).
(c) Yes, since np = 8 and nq = 8 are each greater than 5.

2. Use complement: 1 - [p(0) + p(1) + p(2) + p(3)] = 1 - .49251 (approx.) or approximately .51 (b) P(x > 3.5) correction factor needed; mean is 3.6; std. dev. is 1.5875 so P(z > -.06) = .5239 (c) np = 3.6 which is less than 5. It is not appropriate to use the standard normal to approximate the binomial if np < 5 or nq < 5.

Confidence Intervals

* Confidence intervals are used to answer with question,

How good is $\hat{p}$ at estimating p?

* Create a confidence interval, $\hat{p} \pm E$ for proportions with level of confidence, $1-\alpha$ (in percentage) and interpretation, X% confident that the interval contains the true value. The interval is constructed under the assumption that the sample is random.

* For population proportion: Must meet the conditions for a normal approximation to the binomial, mainly $n\hat{p} \geq 5$ and $n\hat{q} \geq 5$, then:

$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$, where $\hat{p} = \frac{x}{n}$ and $\hat{q} = 1 - \hat{p}$.

For the sample size, solve for n and get: $n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$.

Example Problems:

1. The school board is interested in the proportion of middle school students who do not live with both biological parents. How many students will need to be surveyed so that the board can be 90% confident in the results, with no more than .05 error?

2. Southern Bell wants to determine the proportion of cars with cellular phones. How many cars must be sampled in order to be 90% confident that the sample proportion is in error by no more than 0.02?

3. To determine the percentage of Democrats in a certain state, a random sample of 2,200 was obtained. Of these 1,328 were found to be Democrats. Create a 90% confidence interval and a 99% confidence interval for p, the true proportion of Democrats. Interpret your results.

4. A school system wants to determine the proportion of students who come from broken homes, those households that have gone through divorce. How many students would need to be randomly selected in order to be 98% confident that the sample proportion is in error by no more than .05? In reality, how many should be in your sample to assure an adequate number given that around 90% of the sample will produce responses?

Answers:

1. Use .5 for the proportion in the formula since no previous information is available for the proportion. n = 271 approximately. In reality, one should use about 300 names and hope for 95% return rate which will give 285 surveys.

2. n = 1691.27 in formula, or about 1692. In reality, use between 1780 and 1800. A 95% return rate on 1780 will yield 1691 subjects.

3. 90%: $.5864 < p < .6208$ (90% confident that the true proportion of Democrats in the state is between .586 and .621) 99%: $.5767 < p < .6305$ (99% confident that the true proportion of Democrats in the state is between .577 and .631) You can be extremely confident that there is a majority of Democrats in this state!

4. n = 543 in equation. Solve: (.9)x = 543 to get 603 students. Sample size in reality should be about 610. A 90% return rate presents other problems. What would these be?

What is Hypothesis Testing?

HYPOTHESIS TESTING is a decision making process by which we analyze a sample in an attempt to distinguish between results that can easily occur and results that are highly unlikely. It is the use of sample statistics to determine if the sample is statistically significantly different from the claim. It is used to determine if there is a significant difference or a significant relationship.

Hypothesis - a claim or statement that something is true

Null hypothesis - the claim or statement of a zero or null difference (or no relationship or association); the null hypothesis is the hypothesis or claim that is directly statistically tested. The null hypothesis always includes "equals."

The process of hypothesis testing begins with a claim or statement that something is believed to be true or that something needs to be tested. This belief or need is the reason for the study. This claim or belief may or may not be a null hypothesis.

The assumptions for the selected hypothesis testing process for proportions MUST be met. . .

1. Independent samples
2. $np \geq 5$ and $nq \geq 5$

Hypothesis Testing Procedure:

1. Null Hypothesis, H_o, statement that there is no difference between a parameter and a value or no difference between/among parameters or no relationship. H_o includes the "=", the claim being "tested" statistically; may or may not include the original claim or hypothesis (which is the reason for collecting the data). The assumptions of the test MUST be met or the results are meaningless!

2. Test Statistic is the value obtained from information from the sample; it is a value based on sample data and used in making the decision to reject or to fail to reject the null hypothesis. For proportions, one sample:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

3. <u>Critical value</u> is a number, based on significance level (α), separating the set of values that is the rejection region from the set of values that is the non-rejection region. It is a value from a probability distribution such as the standard normal distribution or t-distribution. This value is dependent on whether or not the test is a two-tailed $(=)$ or one-tailed test $(\leq, \geq)$.

Note: The only reason to use a one-tailed test is if there has been a pilot study or other evidence to indicate such. The original claim (the reason for collecting the data) determines whether the test is one-tailed or two-tailed.

In addition, a <u>p-value</u> can be used instead of finding the critical value. P-values are available on computer printouts and in the computer program. A p-value is the probability of getting the results using your sample data given that the null is true.

4. <u>Conclusion</u>: ONLY TWO possibilities:

* Reject the null hypothesis (test statistic is in the rejection region) or
* Fail to reject the null hypothesis (test statistic is not in the rejection region).

Remember

* Statistics cannot PROVE anything
* Never "accept" a null hypothesis

5. <u>Inference</u>: Interpret for the particular situation, summarize results.

<u>Two types of errors in hypothesis testing</u>:

α is the probability of rejecting the null hypothesis when the null is TRUE (a Type I error).

β is the probability of failing to reject the null hypothesis when the null is FALSE (a Type II error); $1-\beta$ is the power of the test, the probability of rejecting a false null hypothesis.

True state:	<u>Null true</u>	<u>Null False</u>
Action:		
<u>Reject</u> the null hypothesis	α	Correct
<u>Fail to reject</u> the null hypothesis	Correct	β

<u>Level of significance</u> - the α (alpha level) at which you are testing the null hypothesis.

Maximum Type I error is given as α (alpha). It is a measure of error so the smaller the alpha level stated, the stronger the statement when one rejects the null hypothesis. In rejecting the null hypothesis, use the smallest alpha possible where the null hypothesis is still rejected.

Without the variance and mean of the population, one cannot compute β (beta). Beta is a theoretical concept, giving the probability of the second type error one can have in testing the null hypothesis.

<u>One-tailed tests:</u> All the rejection region is in one tail of the distribution.

<u>Two-tailed tests:</u> Half of the alpha value is in each tail and the rejection region is split, half in the right tail and half in the left tail.

NOTE: Some fields of endeavor state null hypotheses using only "=" and not $\leq$ or $\geq$ statements although they might state alternate hypotheses with an inequality. In these instances one should look at the alternate hypothesis (when given) to determine if the test is one-tailed or two-tailed.

Hypothesis Testing, One Sample: Proportions

* <u>Test Statistic</u> is the value from the sample used in comparison:

$$z = \frac{\widehat{p} - p}{\sqrt{\frac{pq}{n}}}$$

* Check for np ≥ 5 and nq ≥ 5

* Be able to compare the results of hypothesis testing to the related confidence interval.

<u>Example Problems:</u>

1. The President claims that a majority of voters are in favor of his foreign aid program. A random sample of 1000 voters showed 545 who favored the foreign aid program. Is there sufficient evidence to support the President's claim? State the null hypothesis, give the test statistic, use alpha of .005 to determine the appropriate critical value, find the p-value, state your conclusions, and interpret your results.

2. In a local survey, 813 of the 1084 respondents indicated support for a ban on household aerosols. It is believed that more than 70% of the

population supports the ban on household aerosols. Provide appropriate hypothesis testing to respond to the above claim.

3. You suspect that the coin used to begin the local football game is not a fair coin; i.e., that there are too many instances of "heads". (A fair coin would give approximately half "heads" and half "tails") You take the suspected coin and toss it 100 times. You get 65 heads. Is this result significantly higher than should be produced by a fair coin? Provide the null hypothesis, test statistic, p-value, conclusion, inference, and evaluate the experiment.

4. The president of a large company (5,000 employees) is certain that a significant majority of his employees is in favor of the new health plan that will go into effect in January. The labor union (3,000 members) is not so certain that a significant majority of labor union members are in favor of this health plan. A stratified random sample of 300 union members and 200 non-union members (from the remaining 2,000 employees) indicated that 153 union members were in favor of the new health plan and 122 non-union members were in favor of the new health plan. (1) Is there a significant majority of union members in favor of the new health plan? (2) Is there a significant majority of non-union members in favor of the new health plan? (3) Was the president of the company correct in his assumption?

5. It is believed that a significant majority of voters are dissatisfied with the local school system. A random sample of 1400 voters was obtained. Of those samples, 746 stated that they were dissatisfied with the school system.
(a) Evaluate the belief with appropriate hypothesis testing. (b) Find the 95% confidence interval for the sample proportion. (c) Are your results from parts a and b consistent? Explain.

Answers:

1. This is a significant majority problem. H_O: $p \leq .5$, $z = 2.846$ is the test statistic, CV is 2.575, p-value is .0022 (the area in the right tail of the standard normal with a z value of 2.85). Reject the null hypothesis at alpha of .005 (since the test statistic, 2.846, is in the rejection region that begins with the CV of 2.575 and extends to the right). It seems as if the President does have a significant majority in favor of the foreign aid program (with possible error of less than .005).

2. H_O: $p \leq .70$. The test statistics is $z = 3.59$ with p-value of .0001. Reject the null hypothesis. The survey supports the claim that more than 70% of the population supports the ban on household aerosols.

3. H_0: $p \le .5$ (for appearance of heads). Test statistic is z = 3.00, p-value or p-level of .0013. Reject the null hypothesis. The coin seems to be unfair. How the coin is tossed could effect the outcome, etc. Also note that an appropriate alpha level for rejection of this hypothesis is .005 that gives a critical value of 2.575.

4. (1) H_0: $p \le .5$, test statistic of z = .35, p-value of .3632, fail to reject H_0. There is not a significant majority of union members in favor of the new health plan. (2) H_0: $p \le .5$, test statistic of z = 3.11, p-value of .0001 (approximately), reject the H_0. There is a significant majority of non-union members in favor of the new health plan. (3) If both groups are combined to form a group of all employees, there would be 275 out of 500 in favor of the health plan. Both the labor union and the non-union members had 10% of the employees sampled. H_0: $p \le .5$, test statistic of z = 2.24, p-value of .0125, reject H_0. There is a significant majority in favor of the new health plan.

5. (a) $H_0 : p \le .5$; z = 2.46; p-value is .0070; Reject the null hypothesis. There seems to be a significant majority dissatisfied with the school system. (b) $.507 < p < .559$ is the 95% confidence interval. (c) The interval does not contain p = .5 and the null hypothesis was rejected; therefore, the results are consistent.

Hypothesis Testing, Two Samples, Proportion

* Must have the following: $n\hat{p} \ge 5, n\hat{q} \ge 5$ for each sample!

* Test statistic:

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\overline{p}\overline{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \text{ where } \overline{p} = \frac{x_1 + x_2}{n_1 + n_2} \text{ and } \overline{q} = 1 - \overline{p}$$

* Confidence Interval:

$$(\hat{p}_1 - \hat{p}_2) \pm E \text{, where } E = z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

X% confident that the difference in the two proportions is in the Confidence Interval.

If a 95% confidence interval for the difference of two proportions contains zero, then the related null hypothesis should not be rejected.

Example Problems:

1. It is believed that the retention rate in private two-year colleges is higher than the retention rate in public two-year colleges. A random sample of 250 two-year public colleges and of 180 two-year private colleges was obtained. This survey determined that the retention rate was 78% for public colleges and 85% for private colleges. Is there a significant difference in the retention rate of these two types of colleges? Show appropriate hypothesis testing procedures.

2. Among 200 randomly selected females, 36% have "9 to 5" daytime jobs. Among 250 randomly selected males, 42% have "9 to 5" daytime jobs. (a) Test the claim that there is no difference between sexes as to the proportion having "9 to 5" jobs. (b) Give a 99% confidence interval for the difference of proportions. Interpret your findings. (c) Are the two results found (parts a and b) consistent? Explain clearly.

3. It is believed that the proportion of college graduates who vote is different from the proportion of high school graduates who vote. A random sample of voters was taken. It was found that 36% of the 250 high school graduates vote while 40% of the 150 college graduates vote. (1) Is the belief correct? (2) Construct a 95% confidence interval for the difference of the two proportions. Explain how your confidence interval is consistent with your hypothesis testing results.

4. For a survey on labor-force participation rates, a random sample of 300 American women and 250 Canadian women are selected. It is found that 184 of the American women and 148 of the Canadian women are in their respective labor forces. (a) Is there a significant difference between the labor-force participation rates of American and Canadian women? Include an appropriate critical value and the p-value. Show all steps clearly. (b) Find a 95% confidence interval for the difference between the labor-force participation rates of American and Canadian women. Interpret your results.

Answers:

1. To answer the question asked: H_0: $p_1 = p_2$ or H_0: $p_1 - p_2 = 0$. Note $x_1 = 195$ and $x_2 = 153$. The test statistic is $z = -1.83$; C.V. for alpha of .05 is ± 1.96. Fail to reject

the null hypothesis. There is not a significant difference in the retention rates of the two groups. If you had been asked to test the original claim, then the null hypothesis would have been H_o: $p_1 \geq p_2$. One would need to reject the null hypothesis at the alpha of .05 since the C.V. is -1.645. This is not a strong rejection and the interpretation should reflect this. Always use common sense when interpreting the results of a directional hypothesis being rejected at an alpha of .05.

2. (a) H_o: $p_1 - p_2 = 0$, test statistic is z = -1.29, p-value of .197. Fail to reject the null hypothesis. There is not a significant difference in the proportion of males and females who have "9 to 5" type jobs. (b) $-.18 < p_1 - p_2 < .06$. One can be 99% confident that the difference in the proportion of females in "9 to 5" jobs and that of males in "9 to 5" jobs is inside the interval -.18 and .06.
(c) The value 0 (in part a, failed to reject H_o: $p_1 - p_2 = 0$) is inside the confidence interval of numbers (-.18 to .06). The two results are consistent.

3. (1) H_o: $p_{hs} - p_c = 0$. The test statistic is z = -.80. Fail to reject H_o. There is not a significant difference between the proportion of high school graduates who vote and the proportion of college graduates who vote. (2) A 95% confidence interval gives: -.04 ± E or $-.14 < p_{hs} - p_c < .06$. The null hypothesis (there is no difference) was not rejected and the value 0 is in the interval. These results are consistent.

4. Note that the assumption is met, np and nq for both samples are well above 5.
(a) $H_0 : p_1 = p_2$ or $H_0 : p_1 - p_2 = 0$; test statistic is .5093 (note: $\overline{p} = .6036$). Critical value: ± 1.96, p-value is .61. Fail to reject the null hypothesis, there seems to be no significant difference in the two labor-force participation rates. (b) E = .0821, $\hat{p}_1 - \hat{p}_2 = .0213$, interval is
(-.0608, .1034). We are 95% confident that the difference in the rates is between -.0608 and .1034. The results are consistent since the interval contains zero and we failed to reject the null hypothesis of no difference.

Chi Square Models

Assumptions: The data are obtained from a random sample; the expected frequency for each category must be 5 or more.

* <u>Goodness of Fit</u>:

H_o: There is no difference in the observed number and the expected number for each category. (This needs to be put into words of the problem.)

test statistic: $\chi^2 = \sum \frac{(O-E)^2}{E}$ with O = observed and E = expected,

E = np where p is the probability of the given category.

The critical value comes from the Chi Square table, where all the rejection region (all alpha) is in the right tail with k-1 degrees of freedom (k is the number of groups or categories).

* Contingency Tables

(1) Test for Independence:

H_o: The variables are independent. (Put into the words of the problem.)

Contingency Table is constructed.

test statistic: $\chi^2 = \sum \frac{(O-E)^2}{E}$ where

E = (row sum)(column sum)/(grand total)

Critical value comes from the Chi Square table, all the rejection region in the right tail with df = (R-1)(C-1).

(2) Test for Homogeneity of Proportions:

H_o: $p_1 = p_2 = p_3 = \ldots p_k$

or the proportions are not different from those predicted.

test statistic: $\chi^2 = \sum \frac{(O-E)^2}{E}$ where

E = (row sum)(column sum)/(grand total)

Critical value comes from the Chi Square table, all the rejection region in the right tail with df = (R-1)(C-1).

Example Problems:

1. It is a common belief that more fatal car crashes occur on certain days of the week, such as Friday or Saturday. A sample of motor vehicle deaths is randomly selected for a recent year. The numbers of fatalities for the different days of the week are listed below. At the .05 significance level, test the claim that accidents occur with equal frequency on the different days. State the null hypothesis, test statistic, critical value, your conclusion and interpretation.

Day	Sun	Mon	Tues	Wed	Thurs	Fri	Sat
Number of Fatalities	31	20	20	22	22	29	26

2. In a study of drug abuse in a local high school, the school board selected 100 eighth graders, 100 sophomores and 100 seniors randomly

from the respective rolls for each grade. Each student was then asked if they used a particular drug frequently, seldom or never. The data are summarized in the table given below. Is there evidence to suggest that the frequency of drug use is the same across the three different grades? State the null hypothesis, give the test statistic, critical value, conclusion, and interpretation.

Frequency of Drug Use

Grade	Frequently	Seldom	Never	TOTAL
Eighth Grade	15	30	55	
Sophomore	20	35	45	
Senior	25	35	40	
TOTAL				

3. In an experiment on extrasensory perception, subjects were asked to identify the month showing on a calendar in the next room. If the results were as shown, test the claim that months were selected with equal frequencies. Assume a significance level of .05. If it appears that the months were not selected with equal frequencies, is the claim that the subjects have extrasensory perception supported?

Jan	Feb	March	April	May	June	July	August	Sept	Oct	Nov	Dec
23	21	35	31	22	41	12	14	10	26	30	24

4. You suspect that a die is unfair. You roll it 60 times and get the following results:

Number on die	1	2	3	4	5	6
Observed frequency	10	12	14	8	12	4

Determine if the above distribution is significantly different from the expected distribution assuming that the die is fair.

5. Students at Oxford were asked the following question: "I find mathematics challenging but I am able to make a good grade." Is there a difference in the distributions of responses between males and females? Students responded as follows:

	agree	no opinion	disagree	TOTAL
males	75	10	85	170
females	121	8	51	180

Give the null hypothesis, test statistic, critical value at an appropriate alpha level, conclusion, and interpretation.

6. Students were asked to respond to the following statement: "participating in study groups is an effective way to study for some courses." Is there a significant difference in the responses of freshmen and sophomores? Show appropriate hypothesis testing responses.

	Agree	No Opinion	Disagree
Freshmen	34	21	35
Sophomore	54	12	29

7. A pair of dice was rolled 500 times. The sums that occurred were as recorded in the following table. Test whether the dice seem fair based on this data. For example, P(2, 3 or 4) = 1/6 and the sums 2, 3 and 4 occurred a total of 74 times. Since the dice was rolled 500 times, one would expect 83.3 (500 X 1/6 = 83.3) occurrences of rolling a 2, 3 or 4, so 83.3 is the expected value.)

Sum:	(2,3,4)	(5,6)	(7)	(8,9)	(10, 11, 12)
Frequency (Observed):	74	120	83	135	88

Rework this problem using the actual observed values for each sum:

Sum	2	3	4	5	6	7	8	9	10	11	12
Obs.:	12	26	36	58	62	83	102	33	20	9	59

Did you find that testing the die this way was significant? Which way would be the best for determining if a die were fair?

Answers:

1. H_o: $p_s = p_m = p_t = p_w = p_t = p_f = p_s$, test statistic = 4.8352, Critical Value is 12.592, fail to reject the null. There is no significant difference among the frequencies of accidents by day of week.

2. H_o: The frequency of drug use is the same across grade levels. C.V. is 9.488 for 4 df at alpha of .05. Test statistic is 5.5, cannot reject null. The frequency of drug use is not significantly different across the three grades.

3. H_o: $p_j = p_f = p_m = \ldots = p_d$. Test statistic of 39.71, C.V. of 19.675. Reject the null. Months were not selected with equal frequencies. It is highly unlikely to get the above distribution by chance. To truly evaluate this crazy experiment, the month showing on the calendar in the next room would need to be known. Was the month changed or the same for each subject questioned?

4. H_o: Each number on the die is equally likely to appear. Test statistic is 6.4, C.V. is 11.071 for degrees of freedom of 5 at alpha of .05. Fail to reject. The die is not significantly different from results that would be obtained from a fair die.

5. Null hypothesis: There is no difference in response distributions between males and females. Or opinions are independent of sex. Test statistic, chi square with df = 2 is 19.245. Critical value is 10.597 for an alpha of .005. Reject the null hypothesis. There is a significant difference between males and females in their responses. Females tend to agree more than do males with the statement, "I find mathematics challenging but I am able to make a good grade.", while males tend to disagree more than females with the statement. (One can interpret responses by checking the chi square value for each cell.)

6. Null hypothesis: There is no difference between responses given by freshmen and sophomores. Test statistic: 7.4327. CV of Chi Square with df = 2: 5.991 for alpha of .05. Reject the null. The opinions of freshmen and sophomores are significantly different. Freshmen seem to agree less and have no opinion more than the sophomores.

7. Null hypothesis: $p_{2,3,4} = p_7 = p_{10,11,12} = \frac{1}{6}$ and $p_{5,6} = p_{8,9} = 1/4$

Chi Square test statistic = 2.305 with CV at .05, df-4, of 9.488. Fail to reject the null hypothesis. The dice seem fair. Probably a better way would be to record the occurrences of each sum and check. When this is done, there is a significant difference and the dice are shown to be unfair.