Math 112 Quizzes 15 & 16 Pledged______________________

You know the rules. Work these at one sitting. Do not use anything except the reference sheet for this course. Do not discuss these quizzes with anyone.

Quiz 15 (40 possible points)

1. Solve the following: (10 points each)

a) $(x^2+1)\frac{dy}{dx}+3x(y-1)=0$

b) $(2y\sin x\cos x-y+2y^2e^{xy^2})dx=(x-\sin^2 x-4xye^{xy^2})dy$

c) $(xy^2+x)dx=(y+1)\sqrt{6x-x^2}\,dy$

2. Evaluate: (5 points each)

a) $\int\frac{x^2}{(x^2+4x+8)^{3/2}}dx$ b) $\int x\sec^8(3x^2)dx$

Quiz 16 (36 possible points)

1. Solve the following: (10 points each)

a) $\frac{dy}{dx}=\frac{xe^{2x+3y}}{ye^{4x-y}}$

b) $(1-2\cos x)dy+(2y\sin x-\tan x)dx=0$

c) $x\ln x\frac{dy}{dx}+y=xe^x$

2. Evaluate: (3 points each)

a) $\lim_{x\to 0}(\sin(3x))^{3/x}$ b) $\lim_{x\to 0}\frac{Arc\sin\left(\frac{x}{4}\right)}{Arc\tan(x^3)}$

Q15

1. a) $(x^2+1)\frac{dy}{dx} + 3x(y-1) = 0$ Linear

$3xy - 3x$

$\frac{dy}{dx} + \frac{3x}{x^2+1}y = \frac{3x}{x^2+1}$

$e^{\int \frac{3x}{x^2+1}dx}$

$e^{\frac{3}{2}\ln|x^2+1|}$

$e^{\ln(x^2+1)^{3/2}}$

$(x^2+1)^{3/2}$

$(x^2+1)^{3/2}\frac{dy}{dx} + 3x(x^2+1)^{1/2}y = 3x(x^2+1)^{1/2}$

NOTE: $\frac{d}{dx}\left(y\cdot(x^2+1)^{3/2}\right) = y\cdot\frac{3}{\cancel{2}}(x^2+1)^{1/2}\cdot\cancel{2}x$

$+ \frac{dy}{dx}(x^2+1)^{3/2}$

$\frac{d}{dx}y(x^2+1)^{3/2} = 3x(x^2+1)^{1/2}$

$\underbrace{\int \frac{d}{dx}y(x^2+1)^{3/2}} = \int 3x\sqrt{x^2+1}\,dx$

$\hookrightarrow u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\frac{3}{2}\int u^{\frac{1}{2}}du = \frac{\cancel{3}}{\cancel{2}}\cdot\frac{\cancel{2}}{\cancel{3}}u^{\frac{3}{2}} + c$

$y(x^2+1)^{3/2} = (x^2+1)^{3/2} + c$

$y = \frac{(x^2+1)^{3/2}}{(x^2+1)^{3/2}} + \frac{c}{(x^2+1)^{3/2}}$

$y = 1 + \frac{c}{(x^2+1)^{3/2}}$

Alternate

1a) $(x^2+1)\dfrac{dy}{dx} + 3x(y-1) = 0$

$$(x^2+1)\frac{dy}{dx} = 3x(1-y) \qquad \underline{\text{Separable}}$$

$$\int \frac{1}{1-y}\,dy = \int \frac{3x}{x^2+1}\,dx$$

$$-\ln|1-y| = \frac{3}{2}\ln|x^2+1| + C$$

$$+\ln|1-y| + \frac{3}{2}\ln|x^2+1| = C$$

$$\ln|(1-y)(x^2+1)^{3/2}| = C$$

$$(1-y)(x^2+1)^{3/2} = e^C = C$$

$$(x^2+1)^{3/2} - y(x^2+1)^{3/2} = C$$

$$y(x^2+1)^{3/2} = (x^2+1)^{3/2} + C$$

$$y = 1 + \frac{C}{(x^2+1)^{3/2}}$$

Same results!

b) $(2y\sin x\cos x - y + 2y^2 e^{xy^2})dx - (x - \sin^2 x - 4xye^{xy^2})dy = 0$

$$\frac{\partial M}{\partial y} = 2\sin x\cos x - 1 + 2y^4 e^{xy^2} + 4ye^{xy^2} \qquad \frac{\partial N}{\partial x} = -1 + 2\sin x\cos x + 4ye^{xy^2} + 2xy^2 e^{xy^2}$$

Exact

$$u = \int(\underbrace{2y\sin x\cos x}_{} - y + \underbrace{2y^2 e^{xy^2}}_{})dx + f(y) = C$$

$u = \sin x$
$du = \cos x\,dx$
$\int 2yu\,du$
$yu^2 + C$

$u = xy^2$
$du = y^2 dy$
$\int 2e^u du = 2e^u + C = 2e^{xy^2} + C$

$$y\sin^2 x - xy + 2e^{xy^2} + f(y) = C$$

$$u = \int(-x + \sin^2 x + 4xye^{xy^2})dy + g(x) = C$$

$-xy + y\sin^2 x + 2e^{xy^2} + g(x) = C$

$u = xy^2$
$du = 2xy\,dy$
$\int 2e^u = 2e^u + C = 2e^{xy^2} + C$

$$y\sin^2 x - xy + 2e^{xy^2} = C$$

$f(y) = C$
$g(x) = C$

c) $(xy^2+x)\,dx = (y+1)\sqrt{6x-x^2}\,dy$

$x(y^2+1)\,dx$

$$\int \frac{x\,dx}{\sqrt{6x-x^2}} = \int \frac{y+1}{y^2+1}\,dy \qquad \underline{\text{Separable}}$$

$9-(x^2-6x+9)$

$$\int \frac{x}{\sqrt{9-(x-3)^2}}\,dx \Rightarrow 3\,\text{Arcsin}\,\frac{x-3}{3} - \sqrt{9-(x-3)^2} + C$$

$u = x-3 \qquad x = u+3$

$du = dx$

$$\int \frac{u+3}{\sqrt{9-u^2}}\,du = \int \frac{u}{\sqrt{9-u^2}}\,du + 3\int \frac{1}{\sqrt{9-u^2}}$$

$3\,\text{Arcsin}\,\frac{u}{3}$

$w = 9-u^2$

$dw = -2u\,du$

$-\frac{1}{2}dw = u\,du \qquad -\frac{1}{2}\int w^{-\frac{1}{2}} = -w^{\frac{1}{2}} + C$

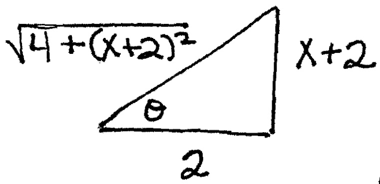
$-\sqrt{9-(x-3)^2} + C$

$$\int \frac{y}{y^2+1}\,dy + \int \frac{1}{y^2+1}\,dy$$

$\frac{1}{2}\ln|y^2+1| + \text{Arctan}\,y + C$

$$3\,\text{Arcsin}\left(\frac{x-3}{3}\right) - \sqrt{9-(x-3)^2} = \frac{1}{2}\ln|y^2+1| + \text{Arctan}\,y + C$$

2a) $\displaystyle\int \frac{x^2\,dx}{(x^2+4x+8)^{3/2}} = \int \frac{x^2\,dx}{[4+(x+2)^2]^{3/2}}$



$\tan\theta = \frac{x+2}{2}$

$x = 2\tan\theta - 2$

$dx = 2\sec^2\theta\,d\theta$

$\theta = \text{Arctan}\left(\frac{x+2}{2}\right)$

$$\int \frac{(2\tan\theta-2)^2 \cdot 2\sec^2\theta\,d\theta}{\underbrace{(4+4\tan^2\theta)^{3/2}}_{\underbrace{(4\sec^2\theta)^{3/2}}_{(2\sec)^3}}}$$

$$\int \frac{\cancel{4}(\tan\theta-1)^2\cancel{2\sec^2\theta}\,d\theta}{\cancel{8}\sec^{\cancel{3}}\theta}$$

$$\int \frac{\tan^2\theta - 2\tan\theta + 1}{\sec\theta}\,d\theta = \int \frac{\sec^2\theta \cancel{-1} - 2\tan\theta \cancel{+1}}{\sec\theta}\,d\theta$$

$$\int \sec\theta - 2\frac{\tan\theta}{\sec\theta}\,d\theta \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\ln|\sec\theta + \tan\theta| - 2\int \frac{\sin\theta}{\cancel{\cos\theta}} \cdot \cancel{\cos\theta}\,d\theta$$

$$+2\cos\theta + C$$

$$\ln\left|\frac{\sqrt{4+(x+2)^2}}{2} + \frac{x+2}{2}\right| + 2\left(\frac{2}{\sqrt{4+(x+2)^2}}\right) + C$$

2b) $\displaystyle\int x\sec^8(3x^2)\,dx = \int x\sec^2(3x^2)\left(1+\tan^2(3x^2)\right)^3 dx$

$u = \tan(3x^2)$

$du = 6x\sec^2(3x^2)\,dx$

$\frac{1}{6}du = x\sec^2(3x^2)\,dx$

$\displaystyle\frac{1}{6}\int (1+u^2)^3\,du$

$\displaystyle\frac{1}{6}\int (1+3u^2+3u^4+u^6)\,du$

$\frac{1}{6}\left(u + u^3 + \frac{3}{5}u^5 + \frac{1}{7}u^7\right) + C$

$\frac{1}{6}\tan(3x^2) + \frac{1}{6}\tan^3(3x^2) + \frac{1}{10}\tan^5(3x^2) + \frac{1}{42}\tan^7(3x^2) + C$

Q16

1a) $\frac{dy}{dx} = \frac{xe^{2x+3y}}{ye^{4x-y}} = \frac{x}{y} e^{2x-4x+3y+y}$

$\frac{dy}{dx} = \frac{x}{y} e^{-2x+4y} = \frac{xe^{-2x} \cdot e^{4y}}{y} = \frac{xe^{-2x}}{ye^{-4y}}$

$\int ye^{4y} dy = \int xe^{-2x} dx$ Separable

$u = y \quad dv = e^{-4y} dy$
$du = dy \quad v = -\frac{1}{4} e^{-4y}$

$u = x \quad dv = e^{-2x} dx$
$du = dx \quad v = -\frac{1}{2} e^{-2x}$

$-\frac{y}{4} e^{-4y} + \frac{1}{4} \int e^{-4y} dy$
$-\frac{1}{16} e^{-4y} + C$

$-\frac{x}{2} e^{-2x} + \int \frac{1}{2} e^{-2x} dx$

$-\frac{y}{4e^{4y}} - \frac{1}{16e^{4y}} + C$

$-\frac{x}{2} e^{-2x} - \frac{1}{4} e^{-2x} + C$

$$-\frac{y}{4e^{4y}} - \frac{1}{16e^{4y}} = -\frac{x}{2e^{2x}} - \frac{1}{4e^{2x}} + C$$

b) $(1-2\cos x) dy + (2y\sin x - \tan x) dx = 0$

$\frac{\partial M}{\partial x} = 2\sin x \qquad \frac{\partial N}{\partial y} = 2\sin x$

Exact

$u = \int (1-2\cos x) dy + g(x) = C$

$= y - 2y\cos x + g(x) = C$

$$u = \int (2y\sin x - \tan x)\,dx + f(y) = C$$

$$-2y\cos x + \ln|\cos x| + f(y) = C$$

$$g(x) = \ln|\cos x| \qquad f(y) = y$$

$$\ln|\cos x| - 2y\cos x + y = C$$

c) $x\ln x \dfrac{dy}{dx} + y = xe^x$ <u>Linear</u>

$$\frac{dy}{dx} + \frac{1}{x\ln x}y = \frac{xe^x}{x\ln x}$$

$$\int \frac{1}{x\ln x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}dx$ $\quad \int \frac{1}{u}\,du$

$\ln|u| + C$

$\ln|\ln x| + C$

$$e^{\int \frac{1}{x\ln x}dx} = e^{\ln|\ln x|}$$
$$= \ln x$$

$$\ln x \frac{dy}{dx} + \frac{\cancel{\ln x}}{x\cancel{\ln x}}y = \frac{\cancel{x}e^x\cancel{\ln x}}{\cancel{x}\cancel{\ln x}}$$

$$\ln x \frac{dy}{dx} + \frac{y}{x} = e^x$$

$$\int \frac{d}{dx}(y\ln x) = \int e^x\,dx$$

$$y\ln x = e^x + C$$

$$y = \frac{e^x}{\ln x} + \frac{C}{\ln x}$$

2.) a) $\lim_{x \to 0} [\sin(3x)]^{3/x}$ DNE

"0^{∞}"

$$\lim_{x \to 0^+} [\sin(3x)]^{\frac{3}{x}} = 0^{+\infty} = 0$$

$$\lim_{x \to 0^-} [\sin(3x)]^{\frac{3}{x}} = 0^{-\infty} = +\infty$$

b) $$\lim_{x \to 0} \frac{\text{Arcsin}\left(\frac{x}{4}\right)}{\text{Arctan}(x^3)} \overset{L'H}{=} \lim_{x \to 0} \frac{\frac{1}{\sqrt{1 - x^2/16}} \cdot \frac{1}{4}}{\frac{1}{1+x^6} \cdot 3x^2}$$

"$\frac{0}{0}$"

$$\overset{Alg}{=} \lim_{x \to 0} \frac{\left(\frac{\cancel{4}}{\sqrt{16 - x^2}} \cdot \frac{1}{\cancel{4}}\right)}{\frac{3x^2}{1+x^6}} \overset{Alg}{=} \lim_{x \to 0} \frac{1}{\sqrt{16-x^2}} \cdot \frac{1+x^6}{3x^2}$$

$$\frac{1}{4 \cdot \underset{\text{positive}}{0}} = \boxed{+\infty}$$